CANADIAN MATHEMATICS EDUCATION STUDY GROUP

41TH ANNUAL MEETING

JUNE 2TH TO JUNE 6TH, 2017

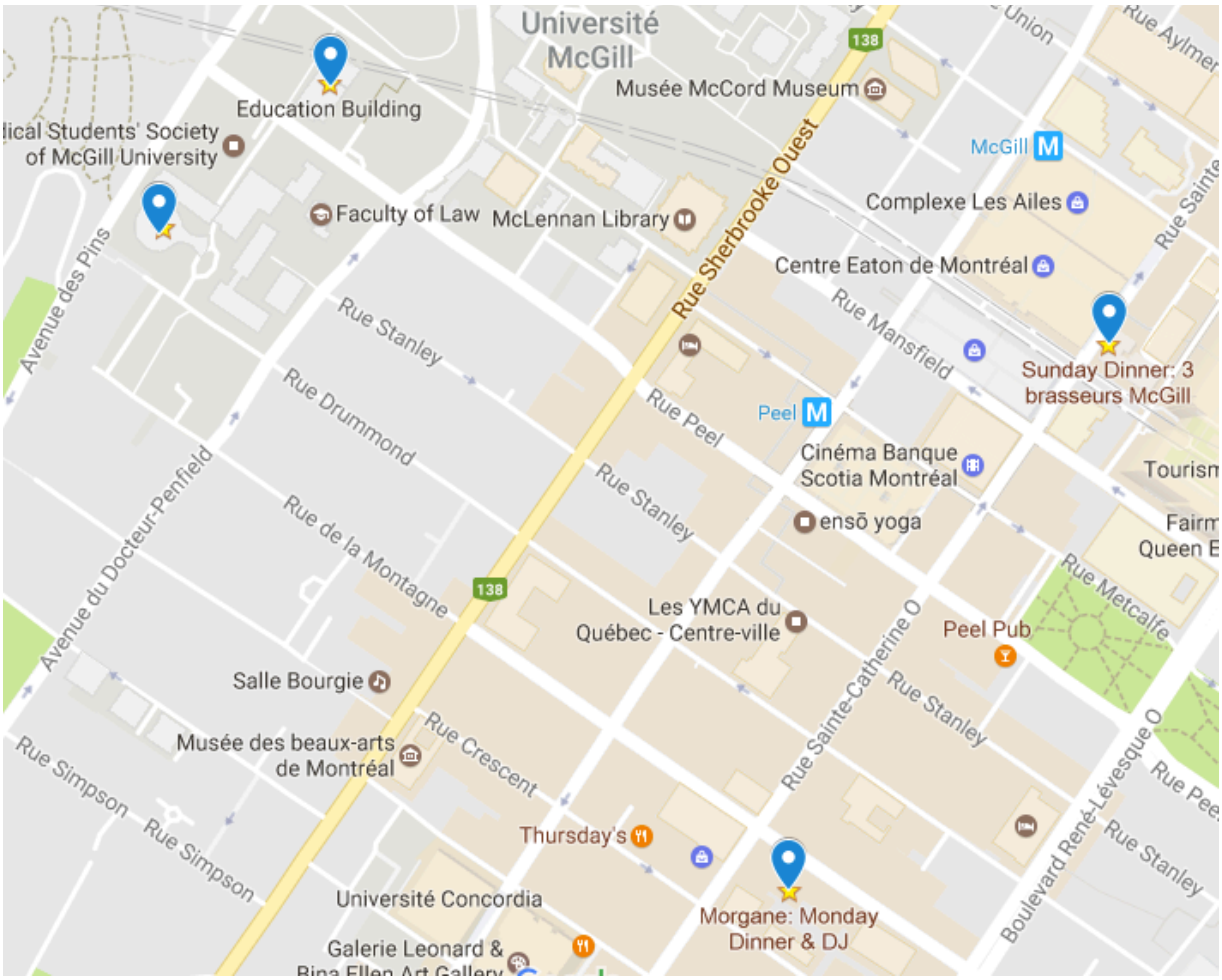
ANNOUNCEMENT AND PROGRAM

We welcome you to McGill University for the 41st Annual Meeting of CMESG/GCEDM, which begins at 6:45 pm on Friday June 2nd and ends at 12:30 pm on Tuesday June 6th. We are especially excited to host this year's Annual Meeting since it coincides with both Montreal's 375th anniversary and Canada's 150th anniversary.

In addition, McGill is hosting the 2017 Colloque du Groupe de didactique des mathématiques du Québec (GDM) from May 31st to June 2nd. We encourage you to also consider attending this conference (http://www.gdm.quebec/prochain-colloque).

Founded in 1821, McGill is the oldest university in Montreal. In addition to McGill's international reputation as a leading academic institution, Montreal has recently been named world's best student city having much to offer to the student and non-student alike.

To learn more about McGill University, you can visit its website http://www.mcgill.ca/ or visit the downtown campus map at http://maps.mcgill.ca/. All on-campus CMESG events will be held either in the Education Building or in the McIntyre Medical Building. Sunday dinner will be held at Les 3 brasseurs McGill (732 St. Catherine Street W., corner McGill College). Monday dinner, DJ, and dancing will happen at the restaurant Morgane de la Montagne (1232 de la Montagne, near St. Catherine Street W.).



WELCOME AND REGISTRATION

Registration on Friday will be from 2:30 pm to 6:45 pm, in the atrium of the McIntyre Medical Building. Dinner (at 5:00 pm) will also be held in the atrium of the McIntyre Medical Building. The opening session (6:15 pm) and the first opening plenary (7:30 pm) will be held in the McIntyre Medical Building (room #522). The reception (8:30 pm) will be held in the atrium of the McIntyre Medical Building.

You will also be able to register between 8:00 am and 9:00 am in the lobby of the Education Building.

HOW TO GET THERE

Montreal is home to Pierre-Elliott Trudeau International Airport, one of the busiest airports in Canada.

From the Airport

- **By taxi:** Rates to downtown are fixed at $40.
- **By bus:** The 747 Express Bus costs $10 to the Lionel-Groulx and Berri-UQAM Métro Station downtown with several stops in between. Tickets can be bought from vending machines inside the airport or on the bus (exact change only, no bills). The ride takes approximately 45 minutes and busses operate 24 hours a day, 7 days a week.

  Note: This $10 ticket is also a **one day bus pass**. Therefore you might want to keep your ticket if you plan on taking a city bus or Metro anytime during the following 24 hours.

- **By Metro**: the closest metro station to the events is Peel Station, on the green line

If you are driving:

- **No right turn on a red light on the island of Montreal!**

By Car

Here are the different routes from the major cities near Montreal:

- *From Sherbrooke*
  - West on Autoroute 10
  - About 150 km, or an hour and 40 minutes
- *From Ottawa*
  - East on Trans-Canada Highway 417, and east on autoroute Transcanadienne 40
  - About 200 km, or about 2 hours
- *From Toronto*
  - East on Highway 401
  - About 540 km, or about 5.5 hours
- *From Québec City*
  - West on autoroute Transcanadienne 40
  - About 250 km, or about 2.75 hours

All on-campus CMESG events will be held either in the Education Building (3700 McTavish Street) or in the McIntyre Medical Centre (3655 promenade Sir William Osler at the top of Drummond Street).

Taking the Métro

- The closest Métro is Peel Station on the Green Line.
- Tickets can be purchased in 1 trip, 2 trips, 10 trips, unlimited evening (6 pm to 5 am), unlimited weekend (Friday 4 pm to Monday 5 am), 1 day, 3 days, or weekly pass (Monday to Sunday) formats. For more information, visit http://www.stm.info/en/info/fares . 1 trip tickets can be purchased aboard a bus with **exact change only**. All other formats must be purchased inside a Métro station. All passes are valid for **both** the metro and buses.

PARKING

For parking at each accommodation, please refer to the Accommodations section.

For on-campus parking, here are the four options closest to the buildings where the conference will be held. For more information on parking zones on the campus, and to get a map of the campus, please visit the following link

https://www.mcgill.ca/transport/parking/downtown/visitors

Education Garage: Monday to Friday $3.50 per 30 minutes to a maximum of $19.00 between 7:00 am and 10:30 pm; $3.50 per 30 minutes to a maximum of $10.00 for entry after 5:00 pm until 10:00 pm. It closes at 8:30 pm on Fridays. Closed Saturday and Sunday. **Accessible with an elevator.**

McIntyre Garage: Monday to Friday $3.50 per 30 minutes to a maximum of $19.00 between 6:00 am and midnight; $3.50 per 30 minutes to a maximum of $10.00 for entry after 5:00 pm until midnight; $3.50 per 30 minutes to a maximum of $10.00 for the overnight period between midnight and 6:00 am. Saturday and Sunday $3.50 per 30 minutes to a maximum of $10.00 between 6:00 am and midnight. **This is a 7-floor garage, without an elevator.**

Drummond lot: Monday to Friday $3.50 per 30 minutes to a maximum of $19.00 between 6:00 am and midnight; $3.50 per 30 minutes to a maximum of $10.00 for entry after 5:00 pm until midnight. Saturday and Sunday $3.50 per 30 minutes to a maximum of $10.00 between 6am and

midnight. **This is an outdoor parking lot and usually fills up quickly, but a good option on weekends.**

You will find a map of the McGill campus and more details on parking areas at the following address: https://www.mcgill.ca/transport/parking/downtown/visitors

ACCOMMODATIONS

We have reserved a block of rooms in one of the McGill University residences. The Carrefour Sherbrooke Residence (475 Sherbrooke West) is conveniently located near campus and downtown Montreal and has double rooms (two queen size beds, work area, private washroom with hair dryer). Standard amenities include: Individual air conditioning control, complimentary wireless internet access, iron and ironing board, TV with cable and remote, in-room telephone, refrigerator, amenities and daily housekeeping. A full American buffet breakfast is also included.

Nightly rates are:
Single and Double occupancy - $115.00
Triple occupancy - $130.00
Quad occupancy - $145.00

*All guest room rates are subject to GST and PST rates in effect at the time of the event. Presently, such taxes are 5% GST, 9.975% PST and 3.5% lodging tax.

To book your accommodation, please contact the McGill University-Accommodations and Conference Services reservations department directly:

Phone: 514-398-5200
Fax: 514-398-4521
E-mail: reserve.residences@mcgill.ca
*Refer to group name: Canadian Mathematics Education or Folio # 475015925

NOTE: Please book your accommodations **as soon as possible** as McGill University-Accommodations and Conference Services has only agreed to hold the full block of guestrooms **until April 25, 2017**.

Airbnb

For those who wish to explore airbnb, consider consulting: www.airbnb.com/montreal

McGill Official Hotels Program

The group of hotels listed below are registered hotels in the McGill Official Hotels Program. To access the discounted rates listed below please make your booking over the telephone and mention McGill University.

Hotel Omni Mont-Royal
1050 rue Sherbrooke St. West
Montreal, QC H3A 2R6
Tel.: 514-284-1110
Website
McGill promotion only available if booking over the telephone.
Price per night: approx. $129 + applicable taxes
Parking $35/day
Wheelchair accessible
Breakfast not included

Le Meridien Versailles
1808 Sherbrooke St. West (south side)
Montreal, QC H3A 1B4
Tel.: 514-933-8111
Website
McGill promotion only available if booking over the telephone.
Price range per night: approx. $165 + applicable taxes
Parking $27/day
Wheelchair accessible
Breakfast not included.

Sofitel Montreal Golden Mile
1155 Sherbrooke Street West
Montreal, QC H3A 2N3
Tel: 514-285-9000
Website
McGill promotion only available if booking over the telephone.
Price range per night: approx. $199 + applicable taxes
For parking and accessibility details please contact hotel directly.
Breakfast not included.

Delta Montreal Hotel
475 President Kennedy Avenue, Montreal, Quebec H3A 1J7
Tel.: 514-286-1986
Website
McGill promotion available when booking over the telephone.
Price range per night: approx. $175 + applicable taxes
Parking $26.50/day
Wheelchair accessible except for Spa entrance
Breakfast not included.

Other hotels near McGill

Prices below may increase as availability decreases.

Hotel Ambrose http://www.hotelambrose.ca/
Price per night: approx. $111 + applicable taxes
Parking $20/day (reservation required)
Breakfast not available. Not wheelchair accessible.

L'appartement Hotel Montreal http://www.appartementhotel.com/en/home.html
Price per night: approx. $169 + applicable taxes
Parking available. Contact hotel for pricing.
For accessibility please contact hotel directly.
Free continental breakfast.

Hotel Best Western Ville-Marie http://www.hotelvillemarie.com/
Price per night: approx. $198 + applicable taxes
Parking available. Contact hotel for pricing.
For accessibility please contact hotel directly.
Breakfast not included.

Holiday Inn Montreal Midtown https://www.ihg.com/holidayinn/hotels/us/en/reservation
Price per night: approx. $253 + applicable taxes
Parking $24.26/day
11 accessible rooms available
Breakfast not included.

Hotel Le Germain Montreal http://www.germainmontreal.com/en/home
Price per night: approx. $260 + applicable taxes
For parking or accessibility details please contact hotel directly.
Free continental breakfast.

Hotel Le St-Martin http://www.lestmartinmontreal.com/
Price per night: approx. $274 + applicable taxes
Parking available. Contact hotel for pricing.
Please contact hotel for accessibility details.
Breakfast not included.

Hilton Garden Inn http://hiltongardeninn3.hilton.com/en/hotels/quebec/hilton-garden-inn-montreal-centre-ville-YULCVGI/index.html
Price per night: approx. $315 + applicable taxes
Parking $26/day
Wheelchair accessible

Residence Inn Montreal Downtown http://www.marriott.com/hotels/travel/yulri-residence-inn-montreal-downtown/
Price per night: approx. $329 + applicable taxes
Offsite parking available at $25/day
Please contact hotel for accessibility details.
Free hot breakfast.

MEALS

All lunches and dinners will be taken with the group, except for dinner on Saturday (dinner on your own) and the dinner on Sunday (we have a reservation at Les 3 Brasseurs, please make sure to confirm your participation on the registration form, everyone is welcome to join). Some accommodations offer breakfast – make sure to check when booking.

EXCURSIONS

We are not organizing any formal excursions this year. Instead, we leave you with some free time to explore Montreal on your own. Here are some of our absolute favorite things to do in Montreal (nos coups de coeur):

For continuously updated information about special activities happening in Montreal, consider downloading the MTL 375 app.

Montreal Botanical Garden
http://espacepourlavie.ca/en/botanical-garden

Musée des Beaux-Arts de Montréal
https://www.mbam.qc.ca/en/

McCord Museum
http://www.musee-mccord.qc.ca/en/

Pointe-à-Callière Museum
https://pacmusee.qc.ca/en/

Canadian Centre for Architecture
http://www.cca.qc.ca/en/

Local Montréal Food Tours (walking food tour)
http://localmontrealtours.com/

Parisian Laundry (Montreal art gallery)
http://www.parisianlaundry.com/en

Place Ville Marie 46th Floor 360° Observation Deck & Exhibits
http://ausommetpvm.com/en/observation-deck/

Montreal Science Centre
http://www.montrealsciencecentre.com/

Mount-Royal Park
Within easy walking distance from McGill. The "interactive map" is especially useful.
http://www.lemontroyal.qc.ca/en/learn-about-mount-royal/homepage.sn

Old Montreal/ Old Port
http://www.vieux.montreal.qc.ca/eng/accueila.htm

Free Montreal Tours and Montreal Food Tours
Free Old Montreal walking tours and local food tours
http://www.freemontrealtours.com/

EMERGENCY

In case of emergency during the conference, you can contact Annie Savard at 613-540-0732 or by email at annie.savard@mcgill.ca. You can also contact Limin Jao by email at limin.jao@mcgill.ca. The University also has a security service available at 514-398-4556; for emergencies call 514-398-3000.

FEES

The conference fee ($210 if registration is received by May 2nd and the full payment before May 9th; $240 thereafter) covers the cost of the reception on Friday, lunches on Saturday, Sunday and Monday, dinners on Friday and Monday, coffee breaks, and other local costs.

The academic program fee is $110 for all participants except full-time graduate students, for whom the fee is $50. This fee is waived for all invited presenters (plenaries, working groups, topic sessions, New PhDs).

Please note: "Ad Hoc" and "Gallery Walk" presenters are required to pay the academic program fee.

ABOUT THE CONFERENCE

CMESG is not a typical academic conference, for it is not organized around presentations and audiences. Instead, it is a conference based on *conferring*.

Its main feature is the **working group**. Each working group will meet for three full mornings to interact around a particular topic. There are two **plenary speaker sessions**, who will each address the whole conference. In contrast with other conferences where questions are often taken at the end of the presentation, a time slot is assigned for the audience, broken into small groups to discuss and prepare questions that will be presented to the speakers in a question period. Two other types of sessions provide more traditional forms of presentation: invited **topic sessions** and the **new PhD sessions**.

Over the course of a meeting (and from meeting to meeting) various discussions and ideas emerge among CMESG members. Our program is designed with time and space for members to come together to work on their emergent ideas. In order to facilitate **Ad Hoc discussions**, there will be a notice board available to request and announce the sessions. Local organizers will assign space for the sessions posted. The nature of the spaces available for ad hoc sessions will reflect the discussion format and the number of sessions proposed. Ad hoc proposers should not expect access to a classroom, computer, projector or power. Hence sessions proposed should be designed with this in mind. There is no reduction in conference fees for presenters in this category. Note— Any person(s) having work prepared in advance to share at the conference should register for the **CMESG Gallery Walk**.

The CMESG Gallery Walk is intended to provide a forum for members to contribute to our meeting and in doing so enhance our awareness of each other's work. We hope this session will increase opportunities for showcasing members' work and building networks among members. We encourage a range of contributions from research posters, to presentations on community initiatives, from mathematics problems, to mathematics art works, anything that can be shared in a gallery format (imagine a poster session or math fair). The session will be broken into two parts allowing every member to participate both as a presenter and as a "walker." One of: a poster board, a piece of the wall, or a table will be provided for each presenter. Presenters will have to supply their own materials and computers (note also, power may not be available). There is no reduction in conference fees for presenters in this category. For more information about this session please contact Olive Chapman at mailto:chapman@ucalgary.ca.

Finally, there is a session that many of us highly value: **meals**! Sit with those you know, sit with those you are getting to know, sit with someone you don't know – the meals are an integral part of the conferring that makes CMESG such a special conference.

FRIENDS OF FOR THE LEARNING OF MATHEMATICS [FLM]

All members of CMESG are also members of the FLM publishing association.

You are invited to meet the FLM journal editor, managing editor and board members at the annual Friends of FLM. This is an informal welcome event organized by the association and an opportunity to learn more about FLM. What makes FLM different? It's the people and more! Everyone invited. Drop by. Refreshments provided.

Friday June 2 15h30 – 16h20. Education Building 233

PLENARY LECTURES

Lecture I ***Yvan Saint-Aubin*** *Université de Montréal, Québec*	***The Most Unglamorous Job of All: Writing Mathematics Exercises***

Most mathematics instructors have chosen their profession because they like to explain and illuminate mathematics, and interact with students. Early in my career of instructor, I saw preparing exercises for my students as a boring (but unavoidable) part of my job. I found quickly that exercises proposed in textbooks are often repetitive and unimaginative, and they fail to address what I think is the crucial point. And then I discovered that writing exercises is difficult: How to formulate problems that will help student understand quickly a new concept, reach some familiarity with it, and discover its raison d'être? The task is solitary, time-consuming and unglamorous, if any. But it can be rewarding!

Lecture II ***Annie Selden*** *New Mexico State University*	***40+ Years of Teaching and Thinking about University Mathematics Students, Proofs, and Proving***

I will briefly describe how my husband and I, who have PhDs in mathematics, got into research in mathematics education. We taught university first in the U.S. and then for 11 years, overseas in Turkey and Nigeria. During this time, we published our first mathematics education paper. In it, we analyzed university students' errors in logical reasoning for a Turkish journal (Selden & Selden, 1978). This was later "recast" in terms of misconceptions for the 1987 *Cornell Misconceptions Conference* (Selden & Selden, 1987).

In 1988, we attended the *Calculus for a New Century Symposium*, held at the National Academy of Sciences, and shortly thereafter, we did a sequence of three small empirical studies on university students' ability to solve non-routine first calculus problems (Selden, Mason, &

Selden, 1989; Selden, Selden, & Mason, 1994; Selden, Selden, Hauk, & Mason, 2000). These will be described.

Subsequently, because we had seen many university students' proving difficulties during our teaching, we switched our research area from university students' difficulties with calculus to their difficulties with proof and proving. The bulk of the talk will be devoted to this work including a description of our "unpacking" and "validation" papers (Selden & Selden, 1995, 2003), which will lead up to a discussion of our more recent theoretical work (Selden & Selden, 2015), including our consideration of the structure of proof texts, as well as our consideration of concepts from the psychological literature.

References:

Selden, A., & Selden, J. (1978) Errors students make in mathematical reasoning. Bosphorus University Journal, 6, 67-87.

Selden, A., & Selden, J. (1987). Errors and misconceptions in college level theorem proving. In J. Novak (Ed.), Proceedings, Second International Seminar on Misconceptions and Educational Strategies in Science and Mathematics (Vol. III, pp. 456-470). Ithaca, NY: Cornell University.

Selden, A., & Selden, J. (2003). Validations of proofs written as texts: Can undergraduates tell whether an argument proves a theorem? Journal for Research in Mathematics Education, 34(1), 4-36.

Selden, A., Selden, J., Hauk, S., & Mason, A. (2000). Why can't calculus students access their knowledge to solve non-routine problems? In A. H. Schoenfeld, J. Kaput, & E. Dubinsky (Eds.), Issues in mathematics education: Vol. 8. Research in collegiate mathematics education. IV (pp. 128-153). Providence, RI: American Mathematical Society.

Selden, J., Mason A., & Selden, A. (1989). Can average calculus students solve nonroutine problems?, Journal of Mathematical Behavior, 8, 45-50.

Selden, J., & Selden A. (2015). A perspective for university students' proof construction. In T. Fukawa-Connelly, N. Infante, K. Keene, & M. Zandieh (Eds.), Proceedings of the 18th Annual Conference on Research in Mathematics Education (pp. 45-59). SIGMAA on RUME. Available online.

Selden, J., & Selden, A. (1995). Unpacking the logic of mathematical statements. Educational Studies in Mathematics, 29(2), 123-151.

Selden, J., Selden, A., & Mason, A. (1994). Even good calculus students can't solve nonroutine problems. In J. Kaput and E. Dubinsky (Eds.), Research issues in undergraduate mathematics learning: Preliminary analyses and results, MAA Notes No. 33 (pp. 19-26). Washington, DC: Mathematical Association of America.

ELDER TALK

Joel Hillel	***Nice, Nice, Very Nice - So Many CMESG People in the Same Device*** ***[with apologies to Kurt Vonnegut Jr. in the Cat's Cradle]***

My transitioning from mathematics to mathematics education coincided with the founding of CMESG/GCEDM so one can say that the organization and my own professional development grew more or less in tandem. In the talk, I'll share some of highlights of this 30-year long symbiotic relationship.

Disclaimer:
Since the talk will likely entail meandering, forgetting, false memories, exaggerations, and possibly, lies, the speaker will bear no responsibility for its content.

CLOSING PANEL

Annette Braconne-Michoux ***Stewart Craven*** ***David Reid*** ***Denis Tanquay*** ***Moderator: Miroslav Lovric***	***Is Mathematics Absolutely Necessary for us to Survive?***

It is truly amazing that it took CMESG 41 years to gain enough courage to put this *most important question* out in the open, to be resolved once and for all!

As mathematicians and mathematics educators where do we stand? Our distinguished panel of experts will try to convince us one way or the other. They will lead a CMESG/GCEDM-style "debate"— friendly and fun! The audience will be invited to contribute through questions and support for the side they favour.

Working Group A *Leaders: Ann Arden, Nadia Hardy and Wes Maciejewski,*	***Teaching First Year Mathematics Courses in Transition from Secondary to Tertiary***

The title of this working group, as proposed by the CMESG organizers, suggests several themes that could be the focus of our time together. The group may choose to focus on the teaching; the transition; the differences and commonalities in content and/or approaches; students' background, motivation, goals, difficulties, misconceptions; teachers' training, motivation, goals; institutional constraints; research frameworks and methodologies. Or perhaps something else?

We, co-leaders hope to bring a diversity of experience: a high school teacher and educator of educators; two mathematicians, one turned education researcher and developmental math coordinator and professor, the other trained as an educational researcher, math professor, and administrator. As such, at the beginning of the first day, we will invite the participants to shape the focus of the working group, in a way that hopefully draws on our and the participants experiences and resources.

Questions we may consider include:

- What is the nature of the mathematics taught and learned in (last year) secondary and (first year) tertiary courses? How is it different and/or similar?
- How do instructional strategies in high school mathematics courses compare with college and university courses? In what ways do secondary and tertiary instructors differ in their approach to main topics of first year courses?
- How does high school teachers' training in teaching mathematics compare to the training of tertiary educators?
- What constraints do teachers and professors have to manage?
- How do assessment strategies in high school mathematics courses compare with college and university courses?
- What effect do the sociological/psychological transitions that students go through as they move from high school to college/university have on their learning and motivation?
- How is the transition from secondary to tertiary mathematics different from the transition from grade 11 to grade 12? Or from first year tertiary courses (e.g., Calculus) to second year tertiary courses (e.g., Analysis)?
- What resources do students have access to and utilize in secondary and tertiary mathematics courses (textbooks, tutorials, videos, technology, etc.)?

- How can we engage/motivate/serve students who are taking minimal math courses ("service courses") in first year? How may this be different from motivating non-math-motivated students through high school?
- What is the nature of remediation courses offered to high school graduates entering college/university and why are these needed more for math courses than other disciplines?
- What theoretical frameworks and methodologies have been proposed for the study of this transition from secondary to tertiary mathematics?

Our overarching goal in this working group is to explore first year tertiary math education in the light of these and other challenges the participants may identify. We invite participants to bring any materials they deem pertinent: syllabi, notes, assessment documents, activities, etc.

References:

The references offered here are food for thought and reflection in preparation for the working group; they don't necessarily reflect what the focus will be.

Artigue, M. (2004, juillet). *Le défi de la transition secondaire/supérieur : Que peuvent nous apporter les recherches didactiques et les innovations développées dans ce domaine.* Communication présentée au 1er Congrès Canada-France des sciences mathématiques, Toulouse.

Clark, M. & Lovric, M. (2008). Suggestion for a theoretical model for secondary-tertiary transition in mathematics. *Mathematics Education Research Journal*. 20(2), 25-37

Clark, M. & Lovric, M. (2009). Understanding secondary-tertiary transition in mathematics. *International Journal of Mathematical Education in Science and Technology*. 40(6), 755-776.

Corriveau, C. & Bednarz, N. (2016a).The secondary-tertiary transition viewed as a change in mathematical cultures: an exploration concerning symbolism and its use. *Educational Studies in Mathematics*. Online: http://link.springer.com/article/10.1007/s10649-016-9738-z

Corriveau, C., et Tanguay, D. (2007). Formalisme accru du secondaire au collégial : les cours d'Algèbre linéaire comme indicateurs. *Bulletin AMQ*, *XLVII*(1), 6-25.

Gueudet, G. (2008). La transition secondaire-supérieur : résultats de recherches didactiques et perspectives. In R. Rouchier (Ed.), *Actes de la XIIIe école d'été de didactique des mathématiques* (CD-Rom). Grenoble: La Pensée Sauvage.

Gueudet, G. (2008). Investigating the secondary-tertiary transition. *Educational Studies in Mathematics*. 67, 237-254.

Hardy, N. (2009). Students' perceptions of institutional practices: The case of limits of functions in college level calculus courses. *Educational Studies in Mathematics*, 72(3), 341-358.

Kajander, A. & Lovric, M. (2005). Transition from secondary to tertiary mathematics: McMaster University experience. *International Journal of Mathematical Education in Science and Technology*. 36(2-3), 149-160.

Luk, H. S. (2005). The gap between secondary school and university mathematics. *International Journal of Mathematical Education in Science and Technology*. 36(2-3), 161-174.

Maciejewski, W. (2016). Instructors' perceptions of their students' conceptions: The case in undergraduate mathematics. *International Journal of Teaching and Learning in Higher Education*. 28(1), 1-8.

Thurston, William. (1994). On proof and progress in mathematics. *Bulletin of the American Mathematical Society*. 30(2), 161-177. https://arxiv.org/pdf/math/9404236v1.pdf

Winsløw, C. (2007). Les problèmes de transition dans l'enseignement de l'analyse et la complémentarité des approches diverses de la didactique. *Annales de didactique et de sciences cognitives, 12*, 195-215.

Working Group B *Leaders: Manon LeBlanc, Jamie Pyper and Jo Towers*	***Elementary Preservice Teachers and Mathematics Anxiety: Searching For New Responses to Enduring Issues***

This Working Group will attempt to bring fresh thinking to the enduring issue of elementary preservice teachers' mathematics anxiety. While this is a topic that has resonated with teacher educators for many years, a recent upsurge of interest in the emotional component of mathematics learning at the Kindergarten to Grade 12 level (e.g., Andersson, Valero, & Meaney, 2015; Brown, Brown, & Bibby, 2008; DiMartino & Zan, 2010; Lange & Meaney, 2011; Takeuchi, Towers, & Plosz, 2016; Towers, Hall, Rapke, Martin, & Andrews, in press; Towers, Takeuchi, Hall, & Martin, 2015) is prompting renewed interest in examining post-secondary students' (and particularly elementary preservice teachers') emotional relationships with mathematics. We feel that it is time to ask again: What motivates, drives, and/or creates elementary preservice teachers' mathematics (teaching) anxiety? And, have these drivers changed recently? Are we as teacher educators relying on (old) assumptions about the nature of our current students' anxieties? Drawing on published and emerging research (e.g., Goulding, Hatch, & Rodd, 2003; Hobden & Mitchell, 2011; Takeuchi, Czuy, & Towers, 2016; Towers,

Takeuchi, Hall, & Martin, in press), we will examine the kinds of K-12 and post-secondary experiences that (may) have led the current generation of math-anxious preservice teachers to their relationships with mathematics, and then move towards explorations of new responses to this enduring problem.

A potential new response is to look closely at the work we do in teacher education that has the deliberate aim of working on preservice teachers' anxieties. We know that our CMESG community houses a wealth of expertise in developing tasks for elementary preservice teacher education that aim to challenge problematic perceptions of mathematics, expand those perceptions, and address students' mathematics anxieties. Part of the work of this Working Group will be to gather, co-develop, play with, and, for our community (and perhaps wider distribution), publish a collection of such tasks. We are aware, though, that exposure to pedagogically rich mathematical tasks is not a panacea for preservice teachers' mathematics anxieties and so we will proceed with caution, inviting Working Group participants to ask: Is it possible to turn all elementary preservice teachers into mathematicians, or even lovers of math, and indeed, should that be our goal?

We will also be inviting Working Group participants to be sensitive to the changing, and ever more political, educational landscape into which our new teachers will step. For example, Ontario has recently moved to emphasize financial literacy in the mathematics curriculum, New Brunswick's recently revised high school mathematics curriculum calls upon learners to choose between three pathways that offer differing levels of applicability to everyday life and careers, and concerns about students' competence in numeracy (rather than mathematics) are emerging in the social discourse in Alberta. In thinking about new responses to the issue of mathematics anxiety among elementary preservice teachers we will explore the curriculum developments/trends that are happening in various provinces and territories with respect to enhancing students' "numeracy" or "mathematical literacy" (rather than mathematics). What effects might such moves have on preservice teachers' math anxieties? Will this ease their fears or add to them? Will preservice teachers feel more, or less, prepared to teach mathematics through such curricula? What are the implications for the work of teacher educators? How can we help new teachers to be more engaged in the broader conversation about the role of mathematics in society?

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Working Group C *Leaders: Judy Larsen, Egan Chernoff, and Viktor Freiman*	***Social Media and Mathematics Education***

The emergence of social media has enabled a dramatic shift in how individuals participate in society. Boundaries such as time and space are defied in ways that make the social media environment capable of producing new manners of interaction. Social media tools allow individuals to participate in the co-creation of publicly available ideas and constructs (e.g., Cardone, 2015; Carpenter & Krutka, 2014; Larsen, 2016). The implications this has on the field of mathematics education are yet to be explored. Mathematicians, mathematics educators, students, and the general public are influencing and are being influenced by publications about the teaching and learning of mathematics made online on various forms of social media (Freiman, 2008). This participatory culture is moving quicker than traditional forms of scholarship and dissemination, and is deserving of attention.

In this working group, key questions relating to the possibilities emerging from social media use and their implications for the field of mathematics education will be asked and discussed. These questions will include:

- What are possible implications of social media use on the teaching and learning of mathematics?
- What are the limitations and repercussions of social media on the authentic representation of broad goals of mathematics education, and how may these barriers be traversed by those interested in advancing the current intersection of social media and mathematics education?
- How are social media knitting the publicly visible landscape of mathematics education, and what are some next steps for capitalizing on this emergent form of public communication?

Participants of this working group will have opportunities to engage in activities that simulate various facets of how social media can be experienced, including examples of mathematical activities that are discussed and curated by mathematics educators on social media platforms, philosophies and information about mathematics education that are commonly shared within social media, and the structure of networks between users. Participant experiences of these activities as well as participants' personal understandings of social media will serve as a backdrop for discussing the working group's guiding questions and related emergent issues. Participants will have opportunities to share their perspectives of how social media and mathematics education intersect. As a group, we will work together to identify issues and

implications arising from the consideration of various perspectives on the connections between social media and mathematics education.

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Working Group D *Leaders: Doris Jeannotte and Lynn McGarvey*	***Quantitative Reasoning in the Early Years***

In the closing panel of 2016 CMESG, Anna Sierpinska questioned whether the kinds of tasks provided to children in elementary school enhanced their quantitative reasoning. In this working group, we would like to extend Anna's wondering by exploring the development of quantitative reasoning in the early years (pre K - 4) from different perspectives. We will draw on recent literature, a range of mathematical tasks, teachers' practice and pupils' responses as part of this exploration.

In particular, we are interested in the links between quantitative reasoning and early algebraic thinking through processes such as generalizing, pattern noticing, conjecturing and justifying.

How are these reasoning processes utilized in tasks involving mathematical structures (e.g., even and odd numbers, commutativity, regularities in multiples, skip counting, adding decimal numbers, etc.)?

Through the working group, we will address the following key questions:

- What is quantitative reasoning and what are the different forms of quantitative reasoning?
- Is early development of quantitative reasoning important and if so, why?
- What types of mathematical tasks emphasize quantitative reasoning processes?
- How might we enhance quantitative reasoning through teaching?

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Working Group E *Leaders: Yasmine Abtahi, Susan Gerofsky, and Jean-François Maheux*	***Social, Cultural, Historical and Philosophical Perspectives on Tools for Mathematics***

In this working group we will examine *tools* for doing mathematics within the social, cultural, historical, theoretical and philosophical contexts of both their use and their origins. By "tools", we mean any of the things people imagine using as part of their mathematical activity! During the working group sessions, we will use a variety of tools to solve mathematical problems, at all educational levels (elementary, secondary and post-secondary), and discuss what takes place in light of different theoretical perspectives. We may explore the 'enabling constraints' of approaching a particular mathematical relationship using historical mathematical tools from cultures ranging from ancient Egypt, medieval Central and South America and Oceania, early modern Europe and China, to contemporary practices. Each kind of tool will be contextualized as much as possible in the philosophical and sociocultural milieu in which it developed. We will draw on Leontiev's ideas on actions, operations and tools; get inspired by Heidegger's work on tools as essentially non-functioning devices; Vygotsky's perspectives on how tools carry with them the perceptions and thoughts of people who made/used/modify them over time and, McLuhan's analysis of the cultural effects of tools and technologies, where we make tools and, immediately, our tools remake us. Participants are welcome to bring their own favourite "mathematical tools": a ruler or slide rule, a phone or abacus, a pencil, an astrolabe, a packet of origami paper or… their fingers!

The mathematical problems we work on will be decided on the first day to fit the interests of the group.

Working Group F *Leaders: Peter Liljedahl, Richelle Marynowski and Sarah Dufour*	***Deep Understanding of School Mathematics***

Ensuring that students develop a 'deep understanding' is a phrase often used to describe the goals of both K-12 and post-secondary mathematics education. If the idea of deep understanding is something we value in mathematics education, it begs the questions:

- What does having a deep understanding of school mathematics mean?
- How is it different from deep understanding of mathematics?
- How can we teach for deep understanding of school mathematics?
- How do we work with pre-service and in-service teachers to teach for deep understanding?

These are open questions within the field of mathematics education. Thus, rather than looking at literature, we will explore these questions through engaging in tasks to provoke conversations and further our understanding of deep understanding. Participation in the working group will stimulate our thinking as researchers, teachers, and teacher educators exploring pedagogical, didactic, curricular, and evaluative implications for practice

TOPIC SESSIONS

Topic Session A *France Caron*	***Modelling Mathematical Modelling***

Mathematical modelling is a powerful way for anticipating or getting insight into real-world situations and phenomena. It thus seems appropriate to approach the integration of modelling in mathematics education from a modelling perspective. In this session, I will start with a representation of the modelling process, more detailed than what we typically see, and I will show how this representation has helped engage discussion with students and professors, design learning activities, analyze student projects and assist students in gaining autonomy with respect to modelling. I will share a recent refinement to this model that has been introduced to reflect in more detail the complexity of simulation-based engineering and help plan a new training program for this specialization. Benefits, limitations and conditions for integrating modelling at the different levels of mathematics education will be discussed with the participants.

Topic Session B *Joyce Mgombelo*	***Collective Learning: Re-thinking the Environment, Artifacts and Classroom Interactions***

Over the past two decades, a group of CMESG members have been working on understanding the ways in which mathematics learning occurs in classrooms viewed as collective systems. That is, how mathematics classrooms can be seen as complex systems in which agents spontaneously interact and adapt to each other, organizing and sustaining learning processes in a collaborative way. In this topic session I will offer a discussion about how artifacts such as a classroom boards play a vital role in the coordination of behaviours or actions in a mathematics classroom viewed as a complex system. The source of inspiration for this discussion comes from studies in cognitive stigmergy which have led to a better understanding of how agents in a complex system communicate indirectly through their environment which is articulated, and is typically composed of artifacts, which build up the social workspace, or field of work.

Topic Session C *John Selden*	***A Psychological View of Teaching Proof Construction***

For more than ten years, Annie Selden and I have co-taught a small experimental course for beginning mathematics graduate students who felt they needed help with proof construction. In the course, students are provided a variety of definitions and theorems, and with some advice, construct their proofs. I will describe some student proving difficulties that we have observed and do so from an easily understood psychological perspective that we are finding useful. I would be very pleased to hear of other experiences/observations similar to (or even opposing) mine.

NEW PHD SESSIONS

Yasmine Abtahi	***Things Kids Think With: The Role Of The Physical Properties Of Mathematical Tools In Children's Learning In The Context Of Addition Of Fractions***

This research was designed to examine the role of the physical properties of the mathematical tools, in children's learning in the context of adding two fractions. My two research questions were: (1) How does the feedback from the mathematical tools play a mediating role between the physical actions of the child with respect to the mathematical affordances of the tools and the child's thinking about and learning and knowing of solving addition of fractions problems? And (2) What role is played by mathematical tools in the emergence of a Zone of Proximal Development during the child's solving of addition of fractions problems? To address these questions, I interviewed 13 grade 7 students in Ottawa, Ontario, in groups of two and in three rounds of 30-minute interviews per group. The results showed that the physical properties of the tools play a role in how children perceived the mathematical affordances of the tool, attached mathematical meaning to the tools, created mathematical artefacts and solved the addition of fractions problems. Moreover, the findings show that in children's interactions with mathematical tools, at times, the Zone of Proximal Development (ZPD) emerged, with the guidance provided by the tools. I conclude that children's interaction with the tools provided them the possibility of learning newer forms of reflections, expressions and actions in relation to adding two fractions. This learning was a result of a complex and intertwined relationship between the immediate physical properties and affordances of the tool, the traces of the thoughts of the designer of the tools, as well as the children's previous knowing of fractions. With this study, I extend the Vygotskian notion of the more knowledgeable other within the ZPD to include not only agents (children and adults) but also tools.

Atinuke Adeyemi	***Examining Mathematics Anxiety Among Classroom Teachers***

Mathematics anxiety affects both teachers and students as it impedes learning and success in mathematics. This sequential mixed methods research investigated the nature and causes of mathematics anxiety among elementary in-service teachers and how the anxiety differs in terms of various demographic factors. It also examined the relationship between mathematics anxiety and mathematics teaching anxiety. Data were collected through an online survey completed by 111 elementary in-service teachers and face-to-face interviews. Findings indicated that participants experienced varied levels of mathematics anxiety; female participants experienced

higher mathematics anxiety than males; and there was a positive correlation between mathematics anxiety and mathematics teaching anxiety. Participants also attributed the causes of mathematics anxiety to their past teachers' teaching strategies and insensitive comments and to themselves. Recommendations are provided on strategies that could be used by teachers, school boards, and teacher educators to reduce mathematics anxiety and break its re-occurring cycle.

Melania Alvarez	***Teaching Teachers: A Look Inside Professional Development***

Extensive research has been conducted on student learning, and pre-service teacher learning inside the classroom, but this is not the case with in-service teachers engaged in learning opportunities provided by professional development. In this study the researcher makes use of the phenomenological perspective, to analyze the 'lived experience' of professional development sessions. To represent the phenomenology of professional development, the researcher developed the idea of scenarios for her analysis. A scenario is defined as a unit of exchange, where the professional developer has a plan, and in accordance with it, s/he introduces or presents an idea or task. This action is taken in by the teachers, and the teachers then respond. The unit is completed when the professional developer takes in the response and sees a need to re-direct. By dividing activities into scenarios, and then focusing on each of its components, the analysis of professional development was considerably simplified.

Jennifer Robin Anderson	***Be Innovative But Don't be Wrong: Are 21st Century Students Experiencing 21st Century Mathematics?***

Using a theoretical framework drawn from appraisal linguistics focussing on interpersonal instances of text, this presentation will report the findings of a study on the judgements made about teaching and learning mathematics and how those judgements reflect the values embedded in mathematics education. As judgement is a highly contextualized enterprise, my presentation will begin with an overview of the current socio-political climate within which teachers are expected to teach and within which students are attempting to succeed. I will also explore current theories that provide a means for researchers to investigate mathematics teaching in context. When outlining the findings, the discussion will be qualitative and interpretive, and I will argue that, despite the fact that the classes were well taught, the students continue a legacy of a narrow concept of the nature and purpose of mathematics and mathematics education.

Carol Carruthers	***College Foundational Mathematics: Can the Affordances of ICT Enhance Self-Regulation Skill of Students?***

This investigation examined the learning gains of an intervention that employed the affordances of information and communication technologies to enhance the self-regulation skills of seventeen students taking a foundational mathematics course at an Ontario community college. The learning intervention consisted of: a) surveys on demographics and perceived ability to self-regulate, b) materials delivered in real time using interactive software and pen-based computing or asynchronously distributed via the learning management system (LMS), and c) student design of studynote (stylus-written MS Word document) or screencast (audio-visual recording) artefacts to demonstrate mathematical solution. Semi-structured interview responses revealed that the creation of these artefacts required goal setting, environment structuring, task strategy planning, and time management skills. When artefacts were viewed using the LMS, individuals compared their work to others (self-evaluation) and sought help if required. These findings indicated that the affordances of the learning intervention contributed to a transformation in self-regulation skill.

Amenda Chow	***Influence of the Learning Environment on Student Test Performance in a Mathematics Course***

It is common practice during an examination to divide students in the same undergraduate class into various locations. Often times, one group of students writes their exam in the lecturing room in which they learned the material, while the remaining students write elsewhere. Due to the familiarity of the learning environment, students writing a test in their lecturing room maybe at an advantage over their peers writing the same test but in a different classroom. This raises concerns about academic fairness. Test scores of engineering students in an undergraduate mathematics course were collected. These results lend insights for training students to use their classroom environment as a mechanism for learning.

Cecilia Kutas Chisu	***The Role of Oral Communication Strategies in Accessing and Assessing Mathematical Understanding***

The study investigated primary teachers' perspectives on teaching mathematics and language arts. It focused on oral communication strategies to build on teachers' greater comfort with teaching language arts. Case studies provided qualitative data through classroom observation, discussions of teaching episodes, semi- formal interviews, and one participant's blog. During the study (2013-2014), school staff worked in partners to teach math through inquiry with an emphasis on communicating mathematical ideas. Evidence collected from grades 1 and 3 lead to several findings that have implications for effective professional development, teachers learning math content and developing teaching materials, improving teacher confidence and the development of mindful reform practice. Suggestions for stakeholders to facilitate teachers' reform practice are included at the end of the study.

Kerry Kwan	***Reciprocal Partnership: An Intervention to Enhance Mathematics Self-efficacy and Achievement of First and Second-semester College Students***

Community colleges are calling to support students who are at risk of restricting their career options because they do not have the mathematical groundings to pursue math-related careers. In response, I conceptualized a mathematics intervention program named, Reciprocal Partnership, which is defined as the collaboration of student dyads to engage in reciprocal learning and teaching under the influence of constructive and collaborative environments that are structured by the Three Learning Situations framework. Reciprocal Partnership is proposed to enrich college students in their mathematical learning and to support their social development during their secondary-tertiary transition for mathematics knowledge and skills are strongly correlated with students' college success and career aspiration. Therefrom, the purpose of this research is to investigate the effect of Reciprocal Partnership on the mathematics self-efficacy and achievement of first- and second-semester college students through a mixed methods approach. It also examines the structure of Reciprocal Partnership to inform the design of effective intervention programs for mathematical learning. Data from final examination grade, and pre- and post-surveys were analyzed through descriptive and inferential statistics, and were used to cross-validate findings from semi-structured interviews. Quantitative results reveal significant effect of Reciprocal Partnership on the mathematics self-efficacy of only students in the first semester, and

no significant effect on the mathematics achievement of both students in the first and second semester. However, qualitative results identify a number of benefits for both groups of students such as gains in mathematical knowledge and skill, confidence, motivation, social connection, and comfort. Findings from this study also suggest the emphasis of mathematics intervention programs on all three learning situations (exploratory, explanatory, and extensional) over only the explanatory situation to maximize learning outcomes.

Terry Wan Jung Lin	***Understanding the Interactions within a New Teacher Learning Community Composed of First Time Participants and a Novice Facilitator***

The notion of a professional learning community (PLC) has been increasingly promoted as a structure to support changes in teaching practices. Most studies present well-established professional communities led by expert facilitators. However, it is unclear how opportunities for learning ambitious teaching practices arise in new communities composed of first time participants and supported by facilitators who are novices in working in this context, and how interactions within the community afford or constrain these opportunities. These are the concerns addressed in this research project. Using sociocultural constructs, I analyzed the dynamics shaping the interactions within a new PLC. The results showed that a wide range of openings to possible learning opportunities arose from the conversations and that these provided windows into the teachers' practice. However, the analysis showed that most interactions following the openings were constraining opportunities for teacher learning because of conflicting cultural models shaping the participants' interactions.

Gale L. Russell	***Valued Kinds of Knowledge and Ways of Knowing in Mathematics and the Teaching and Learning of Mathematics: A Worldview Analysis***

This dissertation is a theoretical investigation of the kinds of knowledge and ways of knowing that are valued within mathematics, and the teaching and learning of mathematics. Using a collage of the methodologies of auto/ethnography, Gadamerian hermeneutics, and grounded theory, mathematics and the teaching and learning of mathematics are analyzed through the lenses of the Traditional Western Worldview and an Indigenous Worldview. In doing this research, I was (and still am) most interested in the points of conflict and tension that exist within

different arenas of mathematics and the teaching and learning of mathematics, how these trouble spots relate to the valuing of different kinds of mathematical knowledge and ways of knowing, and how these issues might be addressed. In addition to proposing a new theory of the teaching and learning of mathematics, this dissertation also proposes a new philosophy of mathematics in support of that theory.

Evan Throop-Robinson	***Open Space Technology: Complexity Thinking, Classroom Discourse, and Mathematics Learning in the Elementary Classroom***

The research focuses on students' discourse in grade 6 mathematics through the intervention of Open Space Technology (OST) (Owen, 1997). The social meeting methodology is claimed to create the conditions for a complex adaptive system to emerge. The research brings together two theoretical perspectives: complexity thinking and discourse theory. Data were generated through five action research cycles and four OST sessions. Classroom discourse analysis (Cazden, 2001) used Sfard's framework for analysis (2007) and Gee's identities building tool (2011). Three types of talk emerged as student conversations showed sequences of meaningful exchanges: (1) sharing information; (2) building knowledge; and, (3) exploring possibilities. Implications for fostering the discourse became apparent as opportunities arose for students to communicate through mathematical words, narratives, routines, and visual mediators. It is suggested that if teachers understand the minimum conditions of complexity thinking they may observe collective emergence through analysis of meaningful exchanges.

Zhaoyun Wang	***Investigating Mathematics Teachers' Knowledge for Teaching and Their Learning Trajectories***

This study investigated three secondary mathematics teachers' knowledge for teaching and their learning trajectories from their own schooling through their establishment as experienced teachers in their education system. Three case studies were conducted through semi-structured research instruments and interviews. Other data such as prior and current official curricula and materials related to teachers' professional development were also collected. The findings indicate that there are five categories of professional knowledge for teachers: subject matter knowledge, curriculum knowledge, knowledge of students, mathematics pedagogy, and knowledge of professional development. Each has its subcategories. The categories and subcategories have their properties and some levels of connections among others. The findings also indicate that the process of teacher professional development is complex. Teachers learned from various formal

and informal sources. Teachers' knowledge for teaching is not static but is dynamic. The knowledge is shaped with the changes of school curriculum and teachers' choice of approaches and learning directions for their professional development.