The Canadian Mathematics Education Study Group

27th Annual Meeting

30 May 3 June 2003

Acadia University

Plenary Lectures

Lecture I
Tom Archibald — Using History of Mathematics in the Classroom: Prospects and Problems

Historical examples have been proposed for use in the mathematics classroom to achieve a variety of aims. In primary and secondary school, the aim may be to give a human context to an otherwise arid subject, or to demonstrate cultural relevance of specific mathematical achievements to a particular group. In the upper years of high school, or in university, historical studies are being used to provide insight into the origin of a particular set of mathematical ideas, or to give insight into why certain results are thought of as important. These and other possible uses are well-represented in a growing literature on the uses of history in the classroom. The recent establishment and rapid growth of the History of Mathematics Special Interest Group of the Mathematical Association of America suggest that, on this continent at least, history is booming as a field of interest among mathematics educators.
In this lecture I propose to give an overview of these recent developments, and then to discuss some of the problems associated with bringing history into the mathematics classroom. Practical issues include but are not limited to: getting bogged down in historical detail at the expense of actually covering the topic the teacher of professor hoped to cover; combining the stultifying boredom of history with the difficulty of mathematics to make the subject even more repellent; and grossly oversimplifying history in order to make some pedagogical point, so that students leave with a manufactured history that is actually incorrect. A more general class of problems arises from the fact that mathematics instructors rarely have much background in the study of history.

Lecture II
Anna Sierpinska — Mathematics education: Teleological considerations

My talk will be a critical reflection on what various authors think research in mathematics education ought to be doing and what research they actually do, based on (and biased by) my reading of some of the published material. The purpose of the talk will be to engage a discussion among the conference participants about the future of our domain.

Working Groups

Working Group A — The History of Mathematics as a pedagogic tool in Grades K-12
Leaders: Louis Charbonneau & Irene Percival

The phrase history of mathematics must surely conjure up different visions to the University researcher studying original texts and the elementary school teacher humanising her math class with anecdotes about mathematicians. Although our sessions will focus on the use of the history of mathematics as a pedagogic tool, rather than as a discipline in its own right, we consider that the researcher has valuable insights to offer practising teachers, and hope to stimulate discussions which will be of interest to those at all levels of education. The precise nature of the working group will depend upon the mix of participants, but we plan to include both large group discussions and smaller groups which will concentrate specifically on teaching at the elementary or secondary level.
Topics will be selected from a variety of mathematical areas. Warm-up activities appropriate to each level will set the scene and provide a focus for reflections about such questions as why should anyone learn about the history of mathematics?, how should the history of mathematics be taught? and to what extent should the history of mathematics be enhanced by connecting it with history in general? David Wheelers question how much should a teacher know about the history of mathematics (CMESG 1981) will be reassessed to determine whether the

intervening twenty years have affected his answer. We obviously feel that some such knowledge is important, and hope that others may be encouraged to try historical approaches in their classrooms.

Working Group B Teacher Research: An Empowering Practice?
Leaders: Louise Poirier, Florence Glanfield & Vicki Zack

The three of us, Vicki, Florence, and Louise, have been involved in practitioner research for a number of years, as teacher-researchers (Vicki), and as academics working collaboratively with primary and secondary teachers and teachers of special-needs children, facilitating and supporting classroom research (Florence and Louise). We invite you to explore a number of aspects with us. We will begin with: What is teacher research? How does one do it? We then aim to go beyond that, dealing with some of the issues posed by Anderson (2002) and Anderson & Herr (1999) recently: Is practitioner research really research? Is practitioner research a separate epistemological entity? Why do practitioner research? Should all teachers do practitioner research? Should faculties of education prepare education practitioners to do education research?

The issues are complex. Some have spoken of the uniqueness, the insider status of the teacher-researcher, the requirement of spiralling self-reflection on action, and the intimate dialectical relationship of research to practice (Anderson & Herr, 1999, p. 12), and have stated that "insider, practitioner research has its own unique set of epistemological, methodological, political and ethical dilemmas " (Anderson, 2002, p. 24). There are issues of ethics, of power, of who's in charge (i. e. whose agenda), as well as of the individual teacher's personal dilemmas and concerns.

We will touch as well upon our work and personal challenges. Louise has been working with colleagues to develop a model of collaborative research. Florence has been working with teachers collecting and analysing research. Vicki has been dealing with the gains as well as the constraints of being a teacher-researcher in the elementary classroom for the past twelve years, for while researching from the inside has been generative and transformative, it has at the same time been very demanding of time and energy.

Anderson, G. (2002). Reflecting on research for doctoral students in education. *Educational Researcher*, *31*(7), pp. 22-25.

Anderson, G. & Herr, K. (1999). The new paradigm wars: Is there room for rigorous practitioner knowledge in schools and universities? *Educational Researcher*, *28*(5), pp. 12-22, p. 40.

Working Group C Images of Undergraduate Mathematics
Leaders: Miroslav Lovric & David Poole

"I hate math!" "What is Fermat's Last Theorem about?" "I really liked your lecture on infinity." "Fractals are cool but I hated those area and perimeter calculations." "Do I have to teach that calculus course again?" "Do all mathematicians look like that guy in Good Will Hunting?" "I have always liked math, and was good at it." "This textbook is useless." "Why do we need all that geometric stuff?" "Who is Sophie Germain?" "Chinese students are expected to do well in mathematics." "What is all this theory good for?" "The problem is that students don't learn that stuff in high school anymore." "Why are you bothering me with questions, just give me the damn answer!" Images, opinions, and views of mathematics are uncountable ... so are emotions and stereotypes. The comments above come from students' comments in course evaluations and journal entries, faculty comments over coffee, and comments in the media. Can you tell which is which?

This working group plans to look at this large and complex space of undergraduate mathematics - to discuss, investigate, and analyze, in an attempt to describe what it looks like. We will not restrict our attention to courses for mathematics majors only: "service" courses will also be considered and we will explore what each type of course can learn from the others.

There are many approaches (historical, cultural, ethnomathematical, teaching/learning, epsilon-delta/no epsilon-delta, etc.), and viewpoints (undergraduate students, university lecturers, high school>

We'll be searching for interesting facts, fresh ideas, and creative insights. What can we learn? Can it help us appreciate mathematics more? Can it lead towards improvements in the way we teach and learn mathematics? What other questions can we possibly ask (and answer?) ?

We propose to organize our time together as follows:
Day 1: Student Images of Undergraduate Mathematics
Day 2: Faculty Images of Undergraduate Mathematics
Day 3: (Re)Imagining Undergraduate Mathematics

Please join us to project, reflect, and transform images of the teaching and learning of undergraduate mathematics.

Topic Groups

Topic Group A The CMS forum on Education
Leader: Malgorzata Dubiel and others

The Canadian Mathematical Society will hold a national educational forum in Montreal in May 2003. This forum will be followed by a second one in Ontario in 2005. These fora will continue the tradition initiated with the first national education forum in Quebec City in May 1995, chaired by Katherine Heinrich. In this topic group participants in the 2003 forum will report their experiences to the CMESG community.

Topic Group B The theory of the six stages of learning with integers
Leader: Zoltan Dienes

One thing that is hard about mathematics is that it isnt about anything. It is just a heap of abstractions, on which a lot of other disciplines are based. So it seems that one hurdle a mathematics educator must get over is the teaching of the process of abstraction.

An abstraction is essentially that which is common to a possibly large number of concrete and observable situations. You could never abstract the concept of dogness from just two or three exemplars of actual dogs. You would have to have a good idea about what is common to a poodle and a great St. Bernard, and yet not common to other animals; having four legs would not be part of the concept, that would be a characteristic sign of quadrupedness.

So one of the first things an educator should do in trying to teach a learner any particular piece of mathematics is to think up a number of concrete situations in which the only common essence has just the properties of the mathematics chosen. Remember the child of four or five who was looking at a picture of ducks who said: I think there are 5 duck in the picture, but there cant be!, and when the teacher asked why the child said: There isnt one in the middle! Clearly this child had not fully abstracted fiveness from her experiences! So this leads to the principle of multiple embodiment.

When we feel that the learner has got hold of the common essence of the various experiences we have provided, he will need to dress it up in some clothes, which will fit all the embodiments. This is the representational stage.

When we feel that for the learner the representation applies to all the embodiments, we can work with the representation itself, which we can then begin to describe, thus developing a symbolic language. Only here do we reach the symbolization stage.

Much later, we can help the learner to organize the symbolic descriptions, putting all the properties described in some logical order. This is the formalization stage, and it is what most professional mathematicians would call real mathematics.

In my talk, I will try to take you through at least some of these stages in the case of the teaching of positive and negative integers. I hope you will find the experience challenging and enjoyable!

Material related to this talk is avaiaible at http://www.zoltandienes.com and in the journal *Mathematics in School*

Topic Group C Exploring the fragility and robustness of students mathematical knowing
Leader: Jo Towers

In ongoing research (with Drs. Simmt and Gordon of the University of Alberta) we are exploring mathematics knowing as it arises in teachers and students actions and interactions. Situating mathematics within our actions and interactions with other humans has prompted us to become interested in the ethical dimensions of teaching and learning mathematics. In this session I will focus on the ways in which teachers and students are placed in relationship with one another through their mathematical ideas.

To explore this relationship I propose to focus our attention on ideas of ‘fragility’ and ‘robustness’ of students’ mathematical knowing. Using as prompts for our thinking videotaped excerpts of elementary school students doing mathematics, we will investigate the emergence of students’ mathematical ideas: Where and how and why do particular mathematical images emerge? What, or who, occasions robust understandings? When are robust images problematic? What role do fragile images play in the growth of understanding? Does (or how does) a focus on fragility and robustness illuminate the relationship between teacher and student?

Topic Group D The Role of Mathematics Contests
Leader: John Grant McLoughlin

Mathematics contests are perceived by some to be written papers that will be used to identify award winners among a relatively elite group of students. While some validity can be granted to such a perception, the role of mathematics contests appears to be misunderstood, in general. The spirit of many existing initiatives places greater emphasis upon offering opportunities for collaboration, broadening the mathematical experiences of students, and developing mathematical abilities at a personal level. This topic group will focus attention on a range of national and regional initiatives, many of which offer models for mathematical community building.

The discussion will extend into consideration of potential benefits for professional development including opportunities for bridging across various levels. Extensive experience including involvement with problem writing teams, seminars with students, and participation in math leagues has shaped personal reflections on this subject. A critical examination of mathematics contests is encouraged as we consider the place of mathematics contests in mathematics education.

Topic Group E Standards for Excellence in Teaching Mathematics

Leader: Steve Thornton

The Australian Association of Mathematics Teachers (AAMT) is the professional organization of teachers of mathematics in Australia. It has 5500 members, almost all of whom are practicing teachers, from levels of schooling ranging from Kindergarten to University. AAMT, and its state-based affiliates, are the principal providers of professional development for teachers of mathematics in Australia, and provide a vehicle through which classroom teachers can provide input into educational directions in Australia.

Mathematics education in Australia has, in recent years, undergone many of the same changes evident in mathematics education throughout the world. Curriculum documents have emphasized mathematics as a creative endeavour, have placed high value on problem-solving and mathematical thinking, and have promoted a technology-rich environment for mathematics learning. Australian-produced teaching resources for mathematics have a high profile, and are well-regarded, both nationally and internationally.

Yet when one looks more closely at the actual practice in mathematics classrooms, it is often dominated by a rule-based, instrumental approach, in which skills take precedence over understanding, and breadth of content takes precedence over depth. Translating the rhetoric into practice remains a critical issue for Australias teachers.

The Australian Association of Mathematics Teachers has, during the past three years, undertaken a major project to publish Standards for Excellence in Teaching Mathematics in Australian Schools. These Standards have been developed using teacher focus groups and teacher work samples, to ensure that they have practical validity and are owned by the profession. The Standards identify characteristics of excellent teachers of mathematics in terms of their professional knowledge, their professional attributes and their professional practice.

The extent to which these Standards can impact upon the day-to-day experiences of children in Australian schools is the principal question facing AAMT in the next few years. It is hoped that the Standards will provide both a framework for future professional development, and a vehicle through which teachers from all levels of schooling can improve their practice and gain accreditation as highly accomplished teachers of mathematics.

This presentation will provide details of the Standards and their development, and will open discussion on the key issue of how professional standards of excellence can be translated into enhanced student learning.

New PhD sessions

France Caron Effects of secondary education on mathematical competencies: an exploratory study

The mathematical preparedness of students entering mathematics-intensive university programs is often perceived as being a problem. This issue calls for studies to better understand the general characteristics of secondary education and its long-term effects on the development of mathematical competencies. In this presentation, we will describe the methodology developed within a doctoral project in which such a study was conducted. During the winter term of 2000, we followed forty Montreal university students in a first-semester course in their engineering, business or computer science program. Mathematical competencies of each of these students were assessed through their performance in solving some of the applied math problems encountered in the course, and possible linkages with individual educational histories were explored. The results obtained suggest avenues for improving secondary mathematics education.

Alex Lawson Documenting the Nature and Degree of Reform Instruction in the Intermediate Classroom

We now have a substantive body of research on the implementation and effect of mathematical reform methods in schools. However, documenting the efficacy of reform remains fraught with problems because the definition of mathematical reform instruction varies widely across studies. Whether a research study determines that a given reform project is effective depends in good part upon what constitutes reform practice for the given study. For some researchers evidence of reform teaching means a greater use of manipulatives and small group work in math instruction. For others, reform instruction means using authentic problems. For some, argument and defence of mathematical thinking is the key reform component.

This study was conducted in part to determine the scope and nature of the implementation of a professional development initiative in Grade 8 reform mathematics instruction. It was also conducted to determine whether a reform model of mathematics instruction, M. Simons (1995; 1997) Mathematics Teaching Cycle, could be adopted, refined and expanded with observation codes in order to more clearly delineate the nature and characteristics of this instruction in the classroom. The model was refined and coded over a series of stages resulting in a research tool

Joyce Mgombelo Mathematics content-pedagogy knowledge: A psychoanalytic and enactivist approach

In my research I used two theoretical frameworks, psychoanalytic and enactivist, to explore the nature of knowledge in mathematics teacher education. In particular I explored the nature and growth of knowledge of mathematics student teachers as they undertake teacher education programs and the possibilities or spaces for growth of this knowledge. Through student teachers narratives the research provides a pedagogical space for mathematics teacher educators to learn about their practice.

Eileen Philips Classroom Explorations of Mathematical Writing With Nine and Ten Year Olds
In this talk about my dissertation (written as a teacherresearcher) I present my longitudinal explorations (19922002) of the area of mathematical and paramathematical writing with grade four pupils (nine- and ten-year-olds) who have been members of my classroom (a public elementary school>model reader and Bakhtins concept of *addressivity* in order to examine larger-scale features of my pupils writing. These connect both to conventional textbook forms and a range of work reported in the research and professional literature, not least under the heading writing to learn mathematics. I coin the term *paramathematical writing*, in order to discuss writing that supports mathematics even though it is not directly mathematical by itself. I identify two forms of paramathematical writing: explicit personal text alongside more overtly mathematical writing and certain syntactic choices (allied to voice) when writing text with the explicit intent of helping another pupil learn some mathematics.

Panel

Nadine Bednarz, Stewart Craven, Donna Karsten and Tom O'Shea
One of the objectives of GCEDM/CMESG is the exchange of ideas and information on all aspects of mathematics teaching in Canada. In this spirit, this round table will deal with curriculum changes in Canada: recent, in progress, or coming soon. Panellists from four different regions of Canada will try to characterize the significant changes on the level of the mathematical curricula of their region, identify challenges from the point of view of teacher education and the implementation of these curricula in the schools, and anticipate, if necessary, limits of these curricula. A discussion will follow during which we will be able to bring forward additional ideas and information, to clarify the possibilities at the national level, and to describe our role as a Canadian mathematics education study group.