CMESG

GCEDM

NEWSLETTER

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The NEWSLETTER is a publication of the
Groupe canadien d'étude en didactique des mathématiques
Canadien Mathematics Education Study Group

CMESG is a group of mathematicians and mathematics educators who meet annually to discuss mathematics education issues at all levels of learning. The aims of the Study Group are:

1) *to study the theories and practices of the teaching of mathematics,*
2) *to promote research in mathematics education,*
3) *to exchange ideas and information about all aspects of mathematics education in Canada,*
4) *to disseminate the results of its work.*

Le GCEDM est composé de personnes oeuvrant en mathématiques et en didactique des mathématiques et qui se réunissent une fois par année pour étudier diverses questions relatives à l'enseignement des mathématiques à tous les niveaux. Les buts du Groupe sont les suivants:

1) *susciter une réflection critique sur la théorie et la pratique de l'enseignement des mathématiques,*
2) *encourager la recherche en didactique des mathématiques,*
3) *faciliter l'échange d'idées et d'informations sur tous les aspects de la didactique des mathématiques au Canada,*
4) *faire connaître les résultats de ses travaux.*

CMESG benefits from ICME-7's financial success

This January CMESG received a cheque for $20,000 from the Executive Committee of ICME-7. The letter expressed the wish that these monies would be used to promote the improvement of mathematics education in Canada, an objective which CMESG embrasses. Special thanks to Claude Gaulin and Bernard Hodgson who did much to

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Le succès financier d'ICME-7 profite au GCEDM

Le Comité exécutif du Congrès ICME-7 a remis en janvier dernier un chèque au montant de 20 000$ au GCEDM. La lettre d'accompagnement exprimait le voeu suivant, qui rejoint clairement les objectifs du GCEDM : « L'Exécutif espère encourager ainsi les actions menées par votre Groupe en vue de promouvoir une

cont. p.12

Minutes of the Annual General Meeting
Canadian Mathematics Education Study Group
York University, 29 May 1993

I. President's Report - Tom Kieren
   A. Duties of the various Executive members for the years 1991-93 were described.
   B. Activities of the Executive were reported: most were in regard to the York meeting and the development of the CMESG Monograph.

II. Secretary-Treasurer's Report - Lars Jansson
   The treasurer reported that income from the monograph, while not separable from other income, was in the neighbourhood of $2,500. This was combined with the Secretary of State grant of $2,200 to cover the expenses related to the monograph. The financial statement of 30 April 1993 gives further financial details.

III. Constitutional Changes
   A. A motion from the Executive regarding a change in sexist language in the constitution had been given notice in the November 1992 Newsletter. This motion, as in the note, was voted upon by membership. CARRIED
   B. The president asked for members to provide advice in regard to the Advisory Board named in the constitution. This Board has not been named in recent years and has seldom functioned. The issue will be turned over to the new Executive.

IV. Proceedings - Tom Kieren and Martyn Quigley
   The 1993 Proceeding will be edited by Martyn. Martyn urged lecturers and group leaders to get their submissions to him promptly.

V. 1994 Meeting
   A. The Executive seeks offers for a place to hold the meeting.
   B. Requests were made for suggestions for speakers and other facets of the next meeting.

VI. Elections - David Wheeler and Lesley Lee
   The winners of the election are: Sandy Dawson and Eric Muller.

VII. Other Business
   ICME-8: Bernard Hodgson requested suggestions regarding the program.

Respectfully submitted: Lars Jansson, Secretary

1995 Meeting

It is not too early to start thinking about the 1995 Annual Meeting. The Executive Committee would welcome your recommendations for the various sessions (names of possible plenary speakers, themes for Working Groups and Topic Groups, etc.) as well as your suggestions about the general structure of the Meeting. Please send these, as soon as possible, to Sandy Dawson (dawson@sfu.ca) or to Eric Muller (emuller@spartan.ac.brocku.ca). Thank you for your interest.

Compte rendu de l'Assemblée générale annuelle
Groupe canadien d'étude en didactique des mathématiques
Université York, 29 mai 1993

I. Rapport du président — Tom Kieren
   A. Le président a d'abord décrit les tâches des membres de l'Exécutif durant la période 1991-93.
   B. Il a ensuite fait rapport sur les activités de l'Exécutif : la plupart concernaient la préparation de la rencontre de York et de la monographie du GCEDM.

II. Rapport du secrétaire-trésorier — Lars Jansson
Les revenus de la vente de la monographie, quoique difficiles à isoler parmi les autres revenus, sont de l'ordre de 2 500$. À cela s'ajoute la subvention de 2 200$ reçue du Secrétariat d'état pour couvrir les dépenses reliées à la monographie. Les états financiers au 30 avril 1993 fournissent plus de détails.

III. Amendements aux Statuts
   A. Le Bulletin de novembre 1992 contenait l'annonce d'une proposition de l'Exécutif concernant la féminisation des Statuts. L'Assemblée est appelée à se prononcer sur cette proposition. ADOPTÉE
   B. Le président demande l'opinion de l'Assemblée au sujet du Comité consultatif dont il est question dans les Statuts. Ce Comité n'a pas été formé depuis plusieurs années et a rarement été actif. La question est renvoyée au prochain Exécutif.

IV. Actes — Tom Kieren et Martyn Quigley
Martyn agira comme éditeur des Actes de 1993. Il prie les conférenciers et responsables de groupes de travail de lui transmettre leurs textes dès que possible.

V. Rencontre de 1994
   A. L'Exécutif demande des propositions d'endroits où pourrait se tenir la rencontre.
   B. Il demande également des suggestions à propos des conférenciers et des autres volets de la rencontre.

VI. Élections — David Wheeler et Lesley Lee
Sont déclarés élus : Sandy Dawson et Eric Muller.

VII. Divers
ICME-8 : Bernard Hodgson demande des suggestions à propos du programme.

Lars Jansson, secrétaire

Rencontre de 1995

Il est déjà temps de penser à la rencontre annuelle de 1995. Le Comité exécutif souhaiterait recevoir vos suggestions quant au programme scientifique (noms de conférenciers et de responsables de groupes de travail, thèmes pour les groupes de travail et les groupes thématiques, etc.) de même que quant à l'organisation générale de la rencontre. Prière de faire parvenir vos commentaires dès que possible à Sandy Dawson (dawson@sfu.ca) ou à Eric Muller (emuller@spartan.ac.brocku.ca). Merci à l'avance de partager avec nous vos bonnes idées !

Notice of Constitutional Amendment

At its meeting in September, the Executive reviewed the role and necessity of the Advisory Board. As there had not been an Advisory Board in place for several years, as the regional representation sought in the Board has been reflected, *de facto*, in the composition of the Executive, and as the Executive proposes the policies of the organization, as well as plans its activities, the Executive is recommending that the Board be discontinued. Therefore,

BE IT RESOLVED THAT:

Paragraphs 2 and 3 in Article 5 of the Constitution and the corresponding guidelines 2-5 be deleted.

As it now stands, Article 5 reads, paragraphs 2 and 3 are in italics

The Executive Committee of the Study Group shall consist of four officers and two members-at-large. The officers shall be a president, two vice-presidents, and a secretary-treasurer. Not all the officers shall be resident in the same province. The Study Group shall establish election procedures to ensure that the composition of the Executive is in accord with the aims and national character of the Study Group, and to govern the appointment of the members of the Executive.

The Advisory Board of the Study Group shall consist of the Executive and eight other members elected annually on a regional basis: two from the western provinces, tw0 from Ontario, two from Quebec, and two from the eastern provinces.

The Advisory Board shall be responsible for setting the general policies of the Study Group and for planning its activities. The Executive Committee shall be responsible for implementing the wishes of the Advisory Board and for the day-to-day management of the Study Group's affairs.

Avis d'amendement aux Statuts

Ayant examiné, lors de sa réunion de septembre dernier, le rôle et la pertinence du Comité consultatif, le Comité exécutif recommande que le Comité consultatif soit aboli. Cette recommandation s'appuie sur les faits suivants : le Comité consultatif est inexistant depuis plusieurs années ; la représentation régionale qu'il est censé refléter est réalisée de fait dans la composition de l'Exécutif : c'est l'Exécutif qui, en pratique, est en charge de la préparation des politiques générales du Groupe d'étude et de la planification des activités. En conséquence,

IL EST PROPOSÉ QUE :

Les 2e et 3e paragraphes de l'Article 5 des Statuts soient abrogés, de même que les Règlements 2 à 5 s'y rapportant.

Le texte actuel de l'Article 5 est le suivant :

Le Comité exécutif du Groupe d'étude comprend quatre personnes en titre, ainsi que deux "membres adjoints". Les personnes en titre consistent en un-e président-e, deux vice-président-e-s et un-e secrétaire trésorier-ère ; elles ne doivent pas toutes résider dans la même province. Le Groupe doit établir une procédure d'élection garantissant que la composition de l'Exécutif soit conforme avec les buts et le caractère national du Groupe, et régissant le mode de désignation des membres de l'Exécutif.

Le Comité consultatif du Groupe d'étude est formé des membres de l'Exécutif et de huit autres membres élus chaque année pour représenter les régions : deux des provinces de l'ouest, deux de l'Ontario, deux du Québec et deux des provinces de l'Atlantique.

Le Comité consultatif est responsable de la préparation des politiques générales du Groupe d'étude et de la planification des activités. Le Comité exécutif est responsable de la mise en oeuvre des recommandations du Comité consultatif et de l'administration quotidienne des affaires du Groupe d'étude.

IN MEMORIAM - NICOLAS HERSCOVICS (1935-1994)

Nous annonçons avec regret le décès du professeur Nicolas Herscovics, du Département de Mathématiques et de Statistiques de l'Université Concordia à Montréal, survenu le 3 janvier dernier, à la veille de ses 59 ans, à la suite d'un long combat contre le cancer.

Le professeur Herscovics était un membre de longue date du Groupe canadien d'étude en didactique des mathématiques. Il a participé à plusieurs de nos rencontres annuelles en y présentant assez souvent des résultats de ses recherches et de ses réflexions. Il avait à coeur l'avancement de nos connaissances en didactique des mathématiques. Aussi ne manquait-il pas la moindre occasion d'inciter ses collègues et ses étudiants à joindre notre groupe. Ses intérêts de recherche ont porté sur l'élaboration de modèles permettant de décrire la compréhension de concepts, modèle pouvant servir de base plus scientifique pour l'évaluation, la remédiation et les interventions pédagogiques. Il ne croyait pas que l'on puisse jamais proposer un modèle de compréhension général pouvant s'appliquer à tous les aspects de l'activité mathématique tels la formation de concept, la résolution de problème, les preuves... C'est la raison pourquoi il a toujours cru qu'il fallait débuter par l'étude de la compréhension de notions particulières. Aussi, trouve-t-on dans ses écrits des analyses du point, de la droite, des petits nombres, des quatre opérations arithmétiques, de la numération positionnelle, de la notion de fraction, de la notion de variable, d'équation, de fonction...

Contrairement à ce que ses écrits pourraient laisser croire, ses préoccupations se voulaient éminemment pratiques: il désirait que le fruit de son travail puisse avoir un impact sur ce qui se passe dans les classes, ce qui ne peut se faire sans passer par les enseignants. Aussi a-t-il éprouvé beaucoup de plaisir à travailler avec eux afin de leur faire acquérir une vision plus globale de ce qu'il appelait "des réseaux de connaissances inter-reliées" qui pourraient combattre une tendance répandue à faire apprendre "des trucs." Le titre de quelques-unes de ses présentations à nos rencontres témoigne de ces intérêts:

It is with regret that we announce the premature death of Nicolas Herscovics, professor in the Department of Mathematics and Statistics at Concordia University, Montreal. He left us on January 3, 1994, on the eve of his 59th birthday, after a long battle with cancer.

Professor Herscovics was a member of the Canadian Mathematics Education Study Group for several years. He participated actively in many of our annual meetings where he liked to discuss results from his research or from his reflections. He wanted to contribute to the advancement of our knowledge in the didactics of mathematics. Therefore, he never missed an occasion to encourage his colleagues and students to join our Group. His main research interest centered on the elaboration of models useful in describing the understanding of concepts, believing that such models could constitute a more scientific basis for evaluation, remediation, and pedagogical intervention. He did not have much credence in the existence of a general model applicable to every aspect of mathematical activity, such as concept formation, problem solving, proofs, and so on--which is why he thought that we should start by describing what understanding means for particular notions. This was the essence of many of his publications which report studies of the notion of point, straight line, natural numbers, numeration, fractions, the four arithmetic operations, variable, and function.

Although at first glance we might be inclined to think that his work was purely theoretical, it is important to mention that fundamentally he aspired to be eminently practical. From the very beginning his ambition was to change what was going on in the classroom. In fact, our first grant was on the integration of research in the preservice and inservice training of teachers, a project that was based on his idea that all innovation must pass through the teacher. He found much pleasure in working with these teachers, helping them to acquire a more global vision of the concepts they were teaching--for him, it was a way to fight their tendency to rely on "recipes." The

1988--Memorial University of Newfoundland: *A cognitive matrix describing the understanding of early multiplication* (Herscovics, Bergeron, Beattys, & Nantais); *A model to describe the construction of mathematical concepts from an epistemological perspective* (Bergeron & Herscovics); *The kindergartners' construction of natural numbers: An international study* (Herscovics & Bergeron).
1989--Brock University: *The construction of natural numbers as seen from an epistemological perspective* (Bergeron & Herscovics).
1990--Simon Fraser University: *Explanatory models of children's mathematics* (groupe de discussion animé avec Bruce Harrison).
1991--University of New Brunswick: *The kindergartners' construction of the number concept* (Herscovics & Bergeron).
Comme son cheminement dans sa vie privée et dans sa vie professionnelle ont déjà été décrits, nous reproduisons ici le texte qui est paru à cet effet dans le Newsletter du groupe international PME (Psychology of Mathematics Education).

Le 3 janvier dernier le PME perdait l'un de ses membres-fondateurs en la personne de Nicolas Herscovics, professeur au Département de Mathématiques et de Statistiques de l'Université Concordia à Montréal, Canada. Lors de sa première participation à une rencontre de CIEM à Karlsruhe en Allemagne en 1976 il trouva que les chercheurs intéressés à l'apprentissage et à l'enseignement des mathématiques avaient tellement à se dire qu'il était regrettable de n'avoir qu'une seule rencontre aux quatre ans. Il demanda alors aux personnes présentes, désireuses de former un sous-groupe de CIEM qui se réunirait tous les ans pour faire avancer la recherche en ce domaine, d'endosser sa proposition. Les signatures recueillies, dont celles de Freudenthal, Fischbein, Karplus, Bauersfeld, et Skemp, furent suffisamment nombreuses et imposantes pour conduire à la création du IGPME (International Group for the Psychology of Mathematics Education)— le premier nom de notre groupe.
Par la suite, Nicolas Herscovics a presque toujours participé aux rencontres annuelles à l'occasion desquelles il faisait beaucoup de

titles of some of his presentations at past CMESG/GCEDM meetings reflect his interests:
1988 meeting at Memorial University of Newfoundland: *A cognitive matrix describing the understanding of early multiplication* (Herscovics, Bergeron, Beattys, & Nantais); *A model to describe the construction of mathematical concepts from an epistemological perspective* (Bergeron & Herscovics); *The kindergartners' construction of natural numbers: An international study* (Herscovics & Bergeron).
1989 meeting at Brock University: *The construction of natural numbers as seen from an epistemological perspective* (Bergeron & Herscovics).
1990 meeting at Simon Fraser University: *Explanatory models of children's mathematics* (A discussion group co-hosted with Bruce Harrison).
1991 meeting at the University of New Brunswick: *The kindergartners' construction of the number concept* (Herscovics & Bergeron).
Other aspects of the personal and professional life of Nicolas Herscovics have been described in a recent memorial text appearing in the Newsletter of the International Group for the Psychology of Mathematics Education (PME). It is reprinted below.

On January 3, 1994, PME lost one of its founding members, Professor Nicolas Herscovics. Nicolas was a faculty member of the Department of Mathematics and Statistics at Concordia University in Montreal, Canada. When he attended his first ICME meeting in Karlsruhe, Germany, in 1976, he deplored the fact that researchers interested in the teaching and learning of mathematics, and who had so much to discuss, were able to gather only once every four years. Therefore, he asked the participants who were interested in forming a subgroup of ICME, one that would meet every year in order to promote research in this domain, to endorse his proposal. The signatures he gathered, such as those of Freudenthal, Fischbein, Karplus, Bauersfeld, and Skemp, were sufficiently numerous and illustrious to lead to the immediate creation of IGPME (International Group for the Psychology of Mathematics Education)--the

travail de coulisse pour inciter des gens à prendre en charge l'organisation d'un colloque dans un pays nouveau afin de briser l'isolement de ses chercheurs et de favoriser de nouvelles vocations. Il a aussi été très actif dans la fondation du PME-NA — le chapitre nord-américain du PME. Avec Carolyn Kieran et moi-même, il a été co-organisateur de deux rencontres annuelles à Montréal, soient le PME-NA V en 1983 et PME-XI en 1987.

Sa carrière de chercheur a été bien brève, soit 15 ans dont les deux dernières au ralenti à cause de sa maladie. En dépit de ce court laps de temps, il a réussi à publier au-delà de soixante-dix publications (pour la plupart accompagnées de communications), dirigé plusieurs maîtrises, co-dirigé trois doctorats avec moi, co-dirigé avec moi un groupe de recherche subventionné pendant toutes ces années par le Ministère de l'Éducation du Québec — toutes ces choses en surplus d'à peu près toujours une pleine charge d'enseignement.

Pendant douze ans il a travaillé avec moi le problème de la compréhension des premières notions mathématiques chez les 5-9 ans. A cet effet nous avons proposé un modèle descriptif de la compréhension qui tienne compte des concepts physiques préliminaires aux notions étudiées. Avec certaines adaptations, ce modèle a pu être appliqué à des notions plus avancées. D'autre part, il s'est aussi intéressé aux problèmes de l'apprentissage et de l'enseignement de l'algèbre qu'il a travaillés avec quelques autres chercheurs. Son dernier intérêt a été la pré-algèbre qu'il a étudiée avec Liora Linchevski d'Israel, et dont il discutait encore tout récemment avec Gerald Goldin.

Un bref regard sur son cheminement de vie explique pourquoi il est arrivé si tard à la recherche. Fils d'un couple juif-roumain immigré en Belgique et qui faisait partie de la résistance durant la dernière guerre mondiale, il fut gardé par un couple catholique à partir du moment où ceux-ci furent envoyés dans des camps de concentration d'où seul son père revint à la fin des hostilités. Celui-ci se remaria, reprit son métier de tailleur et retira de l'école Nicolas, âgé de dix ans, pour lui montrer son propre métier. Lorsqu'il avait seize ans sa famille immigra au Canada où il travailla dans l'industrie du vêtement, puis dans la vente de produits de plomberie tout en

former name of PME.

Thereafter, Nicolas attended almost all of PME's annual meetings where he accomplished much underground work in order to convince colleagues of the necessity of not staying in isolation--one way being to recruit people in their country or even organize a PME meeting. He was also instrumental in founding PME-NA--the North-American Chapter of PME. With Carolyn Kieran and myself, he co-organized two meetings in Montreal: PME-NA V in 1983 and PME-XI in 1987.

His career as a researcher was relatively short, only 15 years--the last two at a reduced pace because of his illness. In spite of such a short period, he nevertheless published over seventy papers (most of them paired with an oral communication), directed many masters degree students, co-directed three doctoral students with me, and co-directed (with me) a research group which was funded by Quebec's Ministry of Education during the last 13 of these 15 years. Concurrently, he almost always had a full teaching load in mathematics and mathematics education!

For twelve years we worked on the problem of describing the understanding of early arithmetic concepts by five-to-nine year olds. We proposed a descriptive model of understanding which was constantly improved in the light of experimental evidence. With minor adaptations it was possible to apply it to more advanced concepts. Nicolas was also interested in the learning and teaching of algebra which he investigated with other colleagues. His last interest centered on pre-algebra which he researched with Liora Linchevski from Israel, and which until very recently, he was discussing with Gerald Goldin.

A brief glance at his life path reveals why he was a latecomer to research. His parents, who were Jewish-Rumanian, migrated to Belgium. During World War II they were arrested and deported to German labor camps, while Nicolas and his twin sister, aged five, were kept by a Catholic couple and also by a nun. In his own words, he was one of the rare Jews who served mass. His mother did not survive the labor camps. After the war, his father returned to Belgium where he married a widow with five sons and returned to his

complétant, par les soirs, ses études secondaires et universitaires. Une fois celles-ci complétées, il abandonna les affaires pour devenir professeur d'université à un salaire deux fois moindre!

Lorsqu'on lui demanda de s'occuper du perfectionnement des maîtres, il sentit le besoin de s'outiller sur les plans pédagogiques et psychologiques et à cette fin il vint chercher à l'Université de Montréal un doctorat en Sciences de l'Éducation qu'il termina en 1979. J'ai eu le bonheur, non pas d'être son directeur de thèse, mais plutôt son accompagnateur. Cette coopération se poursuivit jusqu'en 1991 au moment où se déclara son cancer.

Une douzaine d'années de travail presque quotidien avec lui me permettent de témoigner de sa grande intelligence, de sa créativité, de son esprit d'organisation et de son énergie débordante. Il était très exigeant pour ses collègues et ses étudiants, mais au fond il ne l'était pas plus pour les autres que pour lui-même. Lorsqu'il débattait d'idées, il se révélait sans pitié et sans ménagement pour les sentiments de ses interlocuteurs qui, maintes fois, lui en tenaient rigueur. Par contre, nous l'avons toujours vu, après une période de démolition en règle, devenir très constructif et des plus généreux de son temps et de ses idées. Enfin, il était d'une honnêteté à toute épreuve dans le soin qu'il apportait pour donner crédit aux auteurs de toute idée qu'il utilisait — fût-elles de ses étudiants.

En conclusion, je crois que le monde de la didactique perd un chercheur de qualité qui n'a pu hélas que sillonner notre ciel un court instant. Il m'a souvent dit combien de choses il voudrait faire si la vie lui laissait encore une dizaine d'années. Souhaitons donc que les quelques jalons qu'il a eu le temps de planter puissent indiquer de nouvelles avenues de recherche à d'autres étudiants ou à d'autres collègues.

Jacques C. Bergeron
Département de didactique
Université de Montréal

former trade, tailoring. Nicolas, now aged ten, had to quit school and work in his father's store and learn the trade. When he was sixteen, the entire family moved to Montreal where he worked in the clothing industry and then in the plumbing industry, all the while attending evening school. After graduating from University with a first degree in Physics and a second in Mathematics, he left the business world to become a University Professor--at a substantial cut in salary!

When he was asked to take charge of the In-service program for Secondary level mathematics teachers, he felt the need to acquire some knowledge of Pedagogy and Psychology. So he registered as a Ph.D. student at the University of Montreal, from which he graduated in 1979 at the age of 44. I had the pleasure of being not so much his thesis "director" as his "accompanist." Far from ending with his graduation, our collaboration lasted until 1991 when he learned he had cancer.

A dozen years of almost daily work with him entitle me to attest to his keen intelligence, creativity, organizing ability and inexhaustible energy. He was very demanding of both his students and his collaborators, but it was never more than he required of himself. When debating ideas he could be harsh and without much concern for his opponents' feelings, an attitude that sometimes brought him fierce opposition. Nevertheless, after any such episodes, he always ended up being very positive and most generous with his time and with his ideas. Finally, he was also extremely honest with regard to giving credit for any ideas he used--even when these originated with his students.

In conclusion, I think that the world of mathematical didactics has lost a star, a star that alas crossed our sky for only a short instant. He often told me how much he would have liked to accomplish if only he had another ten or fifteen years left. I can only wish that other researchers or students feel the need and the taste to keep on exploring the new avenues that he traced.

Members Report

The Newsletter welcomes reports on the activities of members of GCEDM/CMESG. This issue includes a detailed account of a research project sent to the Newsletter by Carolyn Kieran. Won't you take a minute or two to drop us a page or so about your research, curriculum development, or teaching actvities?

LEARNING AND TEACHING SECONDARY SCHOOL ALGEBRA: COMPARING PROCESS-ORIENTED AND OBJECT-ORIENTED APPROACHES

Co-investigators: Carolyn Kieran, Université du Québec à Montréal
Anna Sfard, Hebrew University of Jerusalem
Research Assistants: Lesley Lee, Université du Québec à Montréal
Pat Lytle, Université du Québec à Montréal

This year is the first of a three-year, SSHRC-funded, research project in which we are at present teaching two complete Grade 7 classes every day during their regular math period for seven weeks. In brief, our aim is to compare two new and different approaches to the introduction of algebra with computers. While both approaches make use of the unifying characteristics afforded by the concept of function, the first emphasizes the object-like features of graphical representations (using Math Connections Algebra II by Sunburst Wings) whereas the second is grounded in the procedural aspects of numerical calculation processes and tabular representations (using CARAPACE by Boileau and Garançon of UQAM).

Theoretical considerations

Even though there are several possible ways to teach algebra, all of them can be roughly divided into two groups, according to the basic approach they promote: (a) the more traditional approach based on teaching algebraic techniques (such as manipulation of formulae, solving of equations and inequalities, etc.) with no overall conceptual framework--a bottom-up approach in which the unifying concept of function comes late with no real connection to the techniques, and (b) the functional approach in which the unifying concept of function is introduced at the very beginning, from which all algebra is then built up.

On the face of it, the second approach is preferable, as it has the potential of making the learning much more meaningful. Indeed, many teachers, curriculum developers, and researchers seem to take the educational advantages of this approach for granted (e.g., in the ongoing discussion among the leading North American specialists on the learning and teaching of algebra, the question is how to implement this approach rather than whether it is appropriate for the student). The difficulties that students experience with traditional approaches to the teaching of algebra have been amply documented (e.g., Wagner & Kieran, 1989). However, new empirical findings (e.g., Moschkovich, Schoenfeld, & Arcavi, 1993; Sfard & Linchevski, in press) show that the functional approach, if adopted from the very beginning, may be quite difficult for the student. This finding gets its support and explanation from the freshly developed theory of process-object duality of mathematical thinking (Sfard, 1991) which implies that new mathematical objects are not readily accessible to students and may not be well understood until the underlying mathematical processes have been learned and interiorized.

The historical-psychological analysis of different mathematical definitions and representations shows that abstract notions such as number and function can be conceived in two fundamentally different ways: structurally (as objects) or operationally (as processes). The two approaches are contrasted in the following way:

There is a deep ontological gap between operational and structural conceptions. ... Seeing a mathematical entity as an object means being capable of referring to it as if it was a real thing--a static structure, existing somewhere in space and time. It also means being able to recognize the idea "at a glance" and to manipulate it as a whole, without going into details. ... In contrast, interpreting a notion as a process implies regarding it as a potential rather than actual entity, which comes into existence upon request in a sequence of actions. Thus, whereas the structural conception is static, instantaneous, and integrative, the operational is dynamic, sequential, and detailed. (Sfard, 1991, p. 4)

The operational conception is, for most people, the first step in the acquisition of new mathematical notions. For the concept of function, a structural conception might be that of a set of ordered pairs, à la Bourbaki, and an operational conception might include viewing a function as a computational process. Various representations for functions can also be analyzed from a structural/operational perspective:

The computer program clearly corresponds to an operational conception rather than to a structural, since it presents the function as a computational process, not as a unified entity. In the graphic representation, on the other hand, the infinitely many components of the function are combined into a smooth line, so they can be grasped simultaneously as an integrated whole; the graph, therefore, encourages a structural approach. The algebraic representation can easily be interpreted both ways: it may be explained operationally, as a concise description of some computation, or structurally, as a static relation between two magnitudes. (Sfard, 1991, p. 6)

The way in which mathematical concepts, including algebra and the function concept, evolved historically has led Sfard to elaborate a parallel model of mathematical conceptual development in terms of the process-object interpretations of learners (Sfard, 1991). This model can provide a global picture of the demands made on learners by various approaches to the teaching of algebra. One of the built-in aspects of this model is the notion of flexibility in moving from a process interpretation to an object interpretation and vice versa. As Sfard has shown, the development of the long sequence of possible approaches to algebra and to its symbolic constructs took thousands of years. Today, to solve one simple problem from a standard textbook the learner must often resort to all the different perspectives together. The problem solver often has no choice but to oscillate between the operational (process) and structural approach, and between one structural interpretation and another. However, as several studies have suggested, flexibility of perspective is not easy to achieve.

Much of the past research on algebra and functions has not involved graphical representations. Those studies that have are not easily reinterpretable in terms of process-object preferences of students. It is possible that the process-to-object sequence, which has characterized both the historical development of function with respect to its algebraic representation and the empirical support for this sequence as manifested among algebra learners, may be reversed for graphical representations. Sfard herself has stated that "geometric ideas for which the unifying, static graphical representations appear to be more natural than any other, can probably be conceived structurally [as objects] even before full awareness of the alternative procedural [process] descriptions has been achieved" (1991, p. 10). Thus, the following remain open research questions that are central to our study: Is it in the power of graphical tools to change the process-to-object sequence implied by the theoretical framework and reinforced by empirical observations of students? Do students, in the face of an object-oriented teaching approach

supported by the massive use of graphical representations display the same need for process-oriented explanations as they did in other studies when the object-oriented approach was not reinforced by visual means? Of the two teaching approaches--process-oriented and object-oriented--is one more conducive than the other for learning how to shift from either a process perspective to an object perspective or vice versa, according to the demands of the task at hand?

Methodology

The first step in this year's study was to prepare the teaching sequences and materials--one set for the object-oriented approach and another for the process-oriented approach. Pedagogical considerations that entered into the construction of these sequences were the following:

(a) Each sequence would consist of a number of units. Each unit would begin with a teacher's introduction, followed by a series of activities on which the students would work (sometimes on the computer and sometimes off), and concluded by classroom discussion of the concepts touched upon in the unit and their relation with those of the previous units. Each unit would take several days.
(b) The students would always work in pairs on their activities, and always in the same pairs. They chose their own partners prior to the onset of the study. The theoretical writings of Vygotsky concerning the social construction of knowledge motivated us to promote this kind of peer collaboration in the students' everyday activities, as well as in the coming-together of the entire group for discussion at the end of each unit.

After piloting our materials with four pairs of students from another school and subsequently revising them, we began the main study at a relatively small high school in Montreal (275 students) in February 1994. Since the students at this school begin the study of algebra at the start of Grade 8, we decided to work with Grade 7 classes (approximately 12-13 year olds). There are three Grade 7 classes at the school (15 to 20 students per class) and we asked all of them to write a pretest questionnaire that assessed prior algebraic knowledge and inital preferences for certain functional representations. The two classes that were most closely matched were chosen for the study. One class was to be taught with the process-oriented approach and the other with the object-oriented approach. (The third class will be involved in another aspect of the project to be described later.) Before starting the teaching sequences, we interviewed pairs of students in each of these two classes who had scored at an average level on the pretest. Since we intended to follow at very close range two pairs in each class, we wanted to choose two pairs who were able to work fairly well together and were sufficiently verbal with each other to provide us with the kind of data that would show evidence of their ways of thinking and of the concepts that they were finding difficult.

The day the teaching began in both classes, we arrived with all of our video cameras and microphones. It was a bit overwhelming for the students, not only to see all of this audio-visual equipment surrounding them but also to be under the eye of the camera people plus two or three adult observers! We use two cameras in each class; when the teacher is presenting new material or orchestrating a classroom discussion, one camera is focused on her and the other is on the students. When the students are working in pairs, each camera is focused on one of the two pairs selected for detailed observation. When in the computer lab, which is just next door to the math class, the setup is similar, except that there is a third camera capturing all of the computer screen images of one of these two pairs.

Throughout the seven weeks of the study, there are periodic class tests, as well as interviews once a week with the four pairs selected for close observation. At the end of the seven-week study, in addition to interviews with these pairs, a posttest will be given to the two classes, as well as to the math students in the other grades (who have been in an algebra environment for a much longer period of time). The results of the students of our two project classes will be

compared with the results of those in the other grades who have experienced more traditional algebra instruction.

Future plans

As soon as the seven-week project in the two Grade 7 classes comes to an end in mid-March 1994, a math teacher from the school who has been observing both classes during each day of the study will take one of our two sets of materials and teach it to the third Grade 7 math class. This part of the study will provide us with a unique opportunity to see the ways in which a regular math teacher chooses to adapt one of the two approaches to meet his own needs.

The next phase of the study involves the analysis of the mass of video tapes we are in the process of accumulating. It is hoped that this analysis will yield some answers to the process-object questions we posed at the outset of our research. At the same time, we will also be editing the tapes to arrive at a set of excerpts that can be used in a new in-service course planned for next year. This in-service course will focus on the results of the study and will include hands-on instruction in the use of both pieces of software and in integrating them into the appropriate teaching approach or into a combination of both approaches. In the third year of the study, we will observe and support these teachers as they put into practice in their own classrooms the approaches they learned in the in-service course.

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CMESG benefits from ICME-7's financial success (cont.)

generate substantial funds from The Government of Québec and from the Federal Government thereby greatly contributing the financial success of ICME-7.

Le succès financier d'ICME-7 profite au GCEDM (cont.)

meilleure éducation en mathématiques au Canada. » Des remerciements spéciaux sont dus à Claude Gaulin et à Bernard Hodgson, qui ont joué un rôle important dans l'obtention de sommes substantielles des gouvernements du Québec et d'Ottawa, ce qui a grandement contribué au succès financier d'ICME-7.

UPCOMING MEETINGS

Presented in chronological order

American Educational Research Association [AERA], April 4-8, New Orleans, LA.

72nd National Council of Teachers of Mathematics [NCTM], Research Pre-Session, April 11-12, Indianapolis, IN. The conference itself runs from April 13-16.

GCEDM/CMESG Annual Meeting, June 3-7, University of Regina, Regina, Saskatchewan.

Canadian Society for the History and Philosophy of Mathematics [HPM], June 8-10, 1994, University of Calgary, Calgary, Alberta.

Canadian Mathematical Society [CMS] / Société Mathématique du Canada [SMC], June 11-13, 1994, University of Alberta, Edmonton, Alberta.

Canadian Society for the Study of Education [CSSE], The Learned Societies. June 15-18, 1994, University of Calgary, Calgary, Alberta.

Psychology of Mathematics Education [PME], July 29-August 3, Lisbon, Portugal.

The International Congress of Mathematicians [ICM 94], Aug 3-11, 1994, Zürich, Switzerland.

North American Group for the Psychology of Mathematics Education [PMENA], November 5 - 8, 1994, Baton Rouge, Louisanna.

L'EXÉCUTIF DU GCEDM
CMESG EXECUTIVE
1993 - 1994

The members of the Executive extend an invitation to you to contact us about any item of interest. If you have something you want to suggest, if you have a concern you wish to raise, if you want more information, etc., please let one of us know. In order to be of service to the membership, we need to be aware of what your interests are.

Les membres du Comité exécutif vous invitent à leur faire part de votre point de vue concernant n'importe quel aspect de la vie du GCEDM. Que ce soit pour transmettre suggestions ou commentaires, ou encore pour être mieux informé, n'hésitez pas à entrer en contact avec l'un d'entre nous. En nous faisant connaître vos intérêts, vous nous aidez à mieux vous servir.

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